Monetary Economics (Master) - Problem Set Chapter 1

General remarks:
There are two different sets of questions: review questions and exercises. Review questions are for self-study at home. They serve to highlight what you (should) already have learned in the lecture. The review questions will, therefore, not be discussed in class. The exercises, instead, will. Please try and solve the exercises on your own before the exercise session. The parts on the lectures slides that were mentioned as an exercise are not explicitly mentioned here. Please try these as well. And ask questions if there are any.

Problem set 1 will be discussed in the exercise sessions on May 3rd/4th.

Review questions - slides Ch01

- What are the three functions that define "money?"
- Why use general equilibrium models in monetary economics?
- Explain the concept of a "representative agent" (representative household, representative firm)?
- What do we mean by "classical dichotomy?"
- Define the terms "monetary neutrality" and "monetary super-neutrality."
- What is the "Friedman rule?" Explain the rationale behind that rule.
- What is the reason that, in flexible-price models, there are welfare costs of inflation?
- What is the main determinant of these costs?
- Is inflation necessarily always a bad thing?

Exercises

The purpose of the following exercise: derive the first-order conditions in a three-period setting so that

- you practice dynamic optimization in a tractable (though a bit tedious) setting
- you see why the first-order conditions take the form that we had on the board.
- you understand where the transversality condition comes from.

1. A three-period example of the household's problem in the MIU model
Consider a representative household that lives for three periods: $t = 0, 1, 2$. Focus on a setting with perfect foresight. The household maximizes utility by choosing consumption, c_t, real money balances, m_t, and real bond holdings, b_t. In real terms, the household problem is given by:

$$\max_{\{c_t, m_t, b_t\}_{t=0}^2} \sum_{t=0}^{2} \beta^t \left[\ln(c_t) + \eta \ln(m_t)\right], \ \eta > 0, \beta \in (0,1)$$

s.t.

$$c_t + m_t + b_t = w_t + \tau_t + b_{t-1}\frac{1+i_{t-1}}{1+\pi_t} + m_{t-1}\frac{1}{1+\pi_t}, \ t = 0, 1, 2$$

$$m_t \geq 0, \ t = 0, 1, 2$$

$$m_{-1}, b_{-1} \text{ given}$$

The household takes wages, w_t, transfers τ_t, interest rates i_t, and inflation, π_t as given (in all periods). In addition, we need $b_2 \geq 0$.

1. Explain what we mean by a "representative" household.
2. Explain why we need to impose that $b_2 \geq 0$.
3. Write down the Lagrangian as in class.
4. Now write down the Lagrangian without the summation sign (no $\Sigma_{t=0}^2$).
5. Take the first-order conditions with respect to c_0, m_0, b_0, and c_1, m_1, b_1, and c_2, m_2, b_2. Do they look like in class?
6. Look at the final period $t = 2$. Can you find expressions that resemble the transversality conditions?

— end of exercise —

The purpose of the following exercise: derive the first-order conditions under risk in a tractable setting, so that

- you understand what a *state-contingent* choice is, and
- you can see why the expectations terms appear in the first-order conditions on the slides in the way they appear.

2. A two-period example with state-contingent choices.
Consider a representative household that lives for two periods: $t = 0, 1$. There is business cycle risk. In $t = 1$, with probability $prob_H$ the economy will be in a boom. With probability $prob_L = 1 - prob_H$, instead, the economy will be in a recession. Here L marks the recession ("low") and H marks the boom ("high"). Everything dated $t = 0$ is known as of $t = 0$. Due to the business-cycle risk, variables dated $t = 1$ are not known as to $t = 0$, though. Wage income in $t = 1$, for example, is subject to risk. w_1 may take on one of two values: a boom value, w_1^H, or a recession value w_1^L. Similarly, for all the other variables dated $t = 1$.
The household maximizes utility by making choices for c_0, m_0, and b_0, and *state-contingent* choices for c_1, m_1, and b_1. That is, the household may choose different values of consumption, bonds, or money balances depending on whether period $t = 1$ happens to be a boom period or a recession period. In the short-hand notation that we have used on the slides, the household problem is given by

$$\max_{\{c_t, m_t, b_t\}_{t=0}^{1}} \quad E_0 \sum_{t=0}^{1} \beta^t \left[\ln(c_t) + \eta \ln(m_t)\right], \ \eta > 0, \beta \in (0,1).$$

s.t.

$$c_t + m_t + b_t = w_t + \tau_t + b_{t-1}\frac{1+i_{t-1}}{1+\pi_t} + m_{t-1}\frac{1}{1+\pi_t}, \ \ t = 0, 1$$

$$m_t \geq 0, \ t = 0, 1$$

$$m_{-1}, b_{-1} \text{ given },$$

where subscript t now indexes the time period as much as the state of the economy. E_0 marks expectations conditional on all the information available in period 0.

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In more precise notation, the household's problem is:

$$\max_{\{c_0,m_0,b_0,c_1^L,m_1^L,b_1^L,c_1^H,m_1^H,b_1^H\}} \quad [\ln(c_0)+\eta\ln(m_0)]+prob_L\cdot\beta\left[\ln(c_1^L)+\eta\ln(m_1^L)\right]+prob_H\cdot\beta\left[\ln(c_1^H)+\eta\ln(m_1^H)\right]$$

s.t.

$$\begin{array}{l}
c_0+m_0+b_0=w_0+\tau_0+b_{-1}\frac{1+i_{-1}}{1+\pi_0}+m_{-1}\frac{1}{1+\pi_0} \\
m_0\geq 0 \\
c_1^L+m_1^L+b_1^L=w_1^L+\tau_1^L+b_0\frac{1+i_0}{1+\pi_1^L}+m_0\frac{1}{1+\pi_1^L}. \\
m_1^L\geq 0 \\
c_1^H+m_1^H+b_1^H=w_1^H+\tau_1^H+b_0\frac{1+i_0}{1+\pi_1^H}+m_0\frac{1}{1+\pi_1^H}. \\
m_1^H\geq 0 \\
b_1^L\geq 0 \\
b_1^H\geq 0 \\
m_{-1},b_{-1}\text{ given },
\end{array}$$

1. Explain why the household may choose different values for consumption, say, in different states of the world. That is, explain why we allow c_1^H to differ from c_1^L. Why is there a budget constraint for each period t and each state of the world.

2. Write down the Lagrangian.

3. Take the first-order conditions with respect to c_0, b_0, m_0, and c_1^L, m_1^L, b_1^L, and c_1^H, m_1^H, b_1^H.

4. For $t = 0$, do the first-order conditions look like the ones we have on the slides (pages 12 and 14 of slides Ch01_a)?

— end of exercise —

The purpose of the following exercise is:

- to clarify what the Friedman rule means.
- to illustrate that positive inflation targets need not be socially undesirable.
- to illustrate that interest on reserves (or, more literally, interest on outside money) can help implement the social optimum.
- to start thinking about the fiscal implications of monetary policy.
- to realize that the implications of monetary policy typically cannot be judged by looking at one monetary instrument alone. Rather, there typically are many instruments. The mix matters.

3. Interest on money

Consider the following variant of the MIU model discussed in class under certainty. Consider an infinitely-lived representative household $t = 0, 1, 2, ...$ The household maximizes utility by choosing consumption, c_t, real money balances, m_t, and real bond holdings, b_t. In real terms, the household problem is given by:

$$\max_{\{c_t, m_t, b_t\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \beta^t \left[\ln(c_t) + \eta \ln(m_t)\right]$$

s.t.

$$c_t + m_t + b_t = w_t + \tau_t + b_{t-1}\frac{1 + i_{t-1}}{1 + \pi_t} + m_{t-1}\frac{1 + i_t^m}{1 + \pi_t}, \; t = 0, 1, 2, ...$$

$$m_t \geq 0, \; t = 0, 1, 2, ...$$

$$m_{-1}, b_{-1} \text{ given}$$

The household takes wages, w_t, taxes τ_t, interest rates i_t, and inflation, π_t as given (in all periods). Above, i_t^m is interest that that the government pays on money.

1. Explain why we need that, in equilibrium, $i_t^m < i_t$.
2. Write down the first-order conditions of the household.
3. Assume that the government follows a balanced-budget policy. Write down the government budget constraint.
4. Assume that the government follows a constant money growth-rule: $M_t/M_{t-1} = 1 + g_m$. Describe the fiscal implications of changing the growth rate of money, g_m.
5. How do the fiscal implications differ depending on i_t^m?
6. Describe what the optimal monetary policy looks like in this setting. Which combinations of g_m and i_t^m implement the planner's allocation?