Macroeconomics I PhD
Problem Set 6

2022/23

Please hand in at the latest January 25, 2023
https://uni-bonn.sciebo.de/s/mGLWcNHT5nQq5OA

One distinctive characteristic of the Eurozone is that there are multiple fiscal entities, but only one central bank. The following exercises describe this economic arrangement in a stylized way. First, we review a version of the fiscal theory of the price level model studied in class for one country. Thereafter, the model is extended to cover two countries that share a central bank but not a fiscal entity.

1 Fiscal theory of the price level - One country

The economy covers two periods with $t = 0, 1$. The central bank sets the price level of the last period (P_1), such that there is no inflation between the two periods[1]. In period $t = 0$, the central bank sets the interest rate R_0 according to the monetary policy rule

$$R_0 = \frac{1}{\beta} \left(\frac{P_0}{P_{-1}} \right)^\alpha, \tag{1}$$

with $\alpha \geq 0$. P_{-1} is the, in $t = 0$ is the predetermined price level of the period before $t = 0$. There are no costs to price adjustments. The fiscal authority levies taxes and pays down outstanding nominal government debt. We are only concerned with equilibria in which the government repays government debt.

The representative household maximizes the following utility function:

$$\ln(c_0) + \beta \ln(c_1), \ \beta \in (0, 1) \tag{2}$$

while facing the budget contraint in $t = 0$:

$$P_0 c_0 + B_0 + P_0 \tau_0 = P_0 y + \overline{B}_{-1} \overline{R}_{-1} \tag{3}$$

and in $t = 1$:

$$P_1 c_1 + P_1 \tau_1 = P_1 y + B_0 R_0. \tag{4}$$

---

[1] We simply assume that the central bank can do so, without specifying exactly, how the central bank achieves this outcome.

τ_0, τ_1 are real taxes the household pays to the government. y is the exogenously given real endowment with consumption goods households obtain every period. The endowment is constant across periods and not storable. B_0 denotes the nominal government debt bought in $t = 0$, whereas $\overline{B}_{-1}\overline{R}_{-1} > 0$ is a nominal debt obligation of the government in $t = 0$, issued in $t = -1$. Assume on the following that raised taxes are such that $B_0 > 0$. We abstract from all other expenditures of the government besides the ones illustrated above.

a) State the maximization problem of the household and derive the Euler-equation.

b) State the market clearing conditions. **Hint:** After you have substituted the market clearing conditions into the Euler-equation, $1 = \beta R_0/(P_1/P_0)$ holds in equilibrium.

c) Derive the budget constraint of the government in the periods $t = 0$ and $t = 1$.

d) Derive the following equilibrium pricing equation for government debt:

$$\frac{\overline{B}_{-1}\overline{R}_{-1}}{P_0} = \tau_0 + \beta\tau_1 \qquad (5)$$

Give an interpretation of the equation

e) Given the model structure, how would you define passive monetary policy? How would you define active monetary policy? Answer the same question for active and passive fiscal policy.

f) Under which conditions for monetary and fiscal policy is the price level P_0 uniquely determined? When is it indeterminate?

2 Fiscal theory of the price level - Monetary union

The setup remains identical to exercise one, besides having two countries A and B. Both countries have identical sizes and are each populated by one representative household. Both countries have a fiscal authority that issues their own government debt and taxes their domestic population. In period $t = 0$, both countries have nominale debt $B_{-1}^j > 0 \; j \in \{A, B\}$ with the interest rate $\overline{R}_{-1}$.

The central bank sets the interest rate for both countries. Additionally, the central bank controls the price level of the two countries and per assumption ensures $P_1 = P_0$. The fiscal authorities levy taxes and pay down their government debt. Besides their expenses for the downpayment of the government debt, the fiscal authorities do not face other expenditures. To simplify the subsequent calculations, assume that both fiscal authorities only levy taxes in $t = 1$. Moreover, assume that both countries can raise sufficient taxes such as to be able to repay the outstanding debt for every price level.

Each country hosts a representative household. In the following the household problem of the household in country A is described. The problem for the representative household in country B is identical.

The representative household maximizes the utility function

$$\max[\ln(c_0^A) + \beta \ln(c_1^A)], \ \beta \in (0,1) \tag{6}$$

subject to the budget constraint in $t = 0$:

$$P_0 c_0^A + B_0^{A;A} + B_0^{B;A} = P_0 y + \left(\overline{B}_{-1}^{A;A} + \overline{B}_{-1}^{B;A} \right) \overline{R}_{-1} \tag{7}$$

and in $t = 1$:

$$P_1 c_1^A + P_1 \tau_1^A = P_1 y + (B_0^{A;A} + B_0^{B;A}) R_0. \tag{8}$$

where c_t^A is consumption in country A, $B_0^{A;A}$ the government bonds household A ownes of country A, and $B_0^{B;A}$ are the government bonds household A ownes of country B. The identical notation applies to old government debt, where $\overline{B}_{-1}^{A;A} + \overline{B}_{-1}^{A;B} = \overline{B}_{-1}^{A}$, refer to the total outstanding old debt of country A. The endowment with consumption goods of the households in both countries is internationally tradeable. Goods are homogeneous such that the prices of goods in both countries are identical. As before, the endowment $y > 0$ is constant over time, exogenously given and not storable.

a) State the household maximization problem and derive the first order conditions.

b) State the market clearing conditions. Argue that a) and b) imply that in equilibrium $1 = \beta R_0 / (P_1/P_0)$.

c) Derive the budget constraint of the fiscal authority of country A and B in $t = 0$ and $t = 1$.

d) Derive the equilibrium pricing conditions of old government debt $\overline{B}_{-1}^{A}$ and $\overline{B}_{-1}^{B}$ in $t = 0$.

e) Within this economy there exist three players which can act passive or active. Monetary policy of the union, besides the fiscal authorities of both countries. Which combinations of regimes yield a determined price level P_0? When is it indeterminate?

f) The last question is concerned with the welfare implications of monetary unions. Assume that there exists at least one equilibrium price level.

   i) Assume all old debt would be held by the domestic population ($\overline{B}_{-1}^{A,B} = 0$, $\overline{B}_{-1}^{B,A} = 0$). How is the price level determined? Does the price level play a role for the welfare of households A or B? Argue whether your answer depends on the old outstanding government debt ($\overline{B}_{-1}^{A}$, $\overline{B}_{-1}^{B}$ respectively)?

   ii) Assume that $\overline{B}_{-1}^{B;A} = 0.5\overline{B}_{-1}^{B}$, $\overline{B}_{-1}^{A;B} = 0.5\overline{B}_{-1}^{A}$ holds. Hence half of a government's debt is initially owned by the households of the other country. Given this assumption, answer the same questions as in i).