Problem Set 3 – Topics course

Please have your solutions ready by December 9.

1 Uncertainty shocks

Consider the following New Keynesian model. The equilibrium conditions are as follows

Households

Households have GHH preferences in consumption and leisure. Suppose preferences are separable in consumption and leisure. The consumption Euler equation takes the form

$$\lambda_t = \beta E_t \left\{ \lambda_{t+1} R_t / \Pi_{t+1} \right\}.$$

Here λ_t marks the marginal utility of consumption. R_t is the gross nominal interest rate. Π_t the gross inflation rate. $\beta \in (0.1)$ is the time-discount factor.
Marginal utility of consumption is given by

$$\lambda_t = d_t \cdot (c_t - \phi_h h_t^{1+\upsilon}/(1+\upsilon))^{-\omega}.$$

Here c_t is consumption, h_t hours worked. $\phi_h > 0$, $\upsilon \geq 0$, and $\omega > 0$. d_t is a shifter to household time preferences (a "demand shock").
The first-order conditions for labor supply is given by

$$\phi_h h_t^{\upsilon} = w_t,$$

where w_t is the wage.

Firms

The firm sector is the usual New Keynesian mix of intermediate goods producers with sticky prices and a final good producer that bundles the intermediate goods and sells to households.
The New Keynesian Phillips curve is given by

$$\phi_p \Pi_t (\Pi_t - \Pi) = (1-\epsilon) + \epsilon \cdot mc_t + 0.5\phi_p \cdot \epsilon \cdot (\Pi_t - \Pi)^2 + \beta \phi_p E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \frac{y_{t+1}}{y_t} \Pi_{t+1} (\Pi_{t+1} - \Pi) \right\}.$$

Here mc_t marks marginal cost, y_t marks output. Π marks the steady state of inflation. $\phi_p \geq 0$ marks marginal costs. $\epsilon > 1$ marks the elasticity of demand.
Output is produced according to

$$y_t = z_t h_t^{1-\alpha},$$

where $1 - \alpha \in (0, 1]$. z_t is a shock to productivity.
Marginal costs are given by

$$mc_t = \frac{w_t}{1-\alpha} \frac{y_t^{\frac{1}{1-\alpha}-1}}{z_t^{1/(1-\alpha)}}.$$

Monetary policy

The central bank follows an interest-rate rule:

$$R_t = R\left(\frac{R_{t-1}}{R}\right)^{\phi_R}\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\Pi(1-\phi_R)}\left(\frac{y_t}{y}\right)^{\phi_y(1-\phi_R)}\exp\{\sigma_m e_t^m\}.$$

Here R, y mark the steady-state values of the respective variables. $\phi_R \in [0,1)$, $\phi_\Pi > 1$ and $\phi_y \geq 0$. $\sigma_m \geq 0$ is the standard deviation of an iid N(0,1) monetary policy shock e_t^m.

Shocks

Both demand and supply shocks have two components. There are innovations to the levels of the shocks, and to their volatility (shocks to risk). The demand shock evolves according to

$$\log(d_t) = \rho_d \log(d_{t-1}) + \exp\{\sigma_{d,t}\}\exp\{\gamma_d\}e_t^d.$$

Here $\rho_d \in [0,1)$, $\gamma_d \in \mathbb{R}$. e_t^d is an iid N(0,1) innovation. Above, $\sigma_{d,t}$ marks the stochastic volatility of the shock. It evolves according to

$$\sigma_{d,t} = \rho_{\sigma_d}\sigma_{d,t-1} + (1-\rho_{\sigma_d}^2)^{1/2}\eta_d u_t^d,$$

where $\rho_{\sigma_d} \in [0,1)$, $\eta_d \geq 0$. u_t^d is an iid N(0,1) innovation to the volatility.
The productivity shock evolves according to

$$\log(z_t) = \rho_z \log(z_{t-1}) + \exp\{\sigma_{z,t}\}\exp\{\gamma_z\}e_t^z.$$

Here $\rho_z \in [0,1)$, $\gamma_z \in \mathbb{R}$. e_t^z is an iid N(0,1) innovation. Above, $\sigma_{z,t}$ marks the stochastic volatility of the shock. It evolves according to

$$\sigma_{z,t} = \rho_{\sigma_z}\sigma_{z,t-1} + (1-\rho_{\sigma_z}^2)^{1/2}\eta_z u_t^z,$$

where $\rho_{\sigma_z} \in [0,1)$, $\eta_z \geq 0$. u_t^z is an iid N(0,1) innovation to the volatility.

Market clearing

In equilibrium

$$y_t = c_t.$$

(We abstract here from the costs of price adjustment, to keep it simple).

Targets and parameter values

Set up the model to run at monthly frequency. Target a steady state with two percent inflation per year and a nominal rate of three percent per year. In addition, target $h = 1/3$.

Next, let $\omega = 1$, $\upsilon = 0.5$, $\epsilon = 11$, $\alpha = 0.3$, $\phi_p = 1310$, $\phi_R = 0.7^{1/3}$, $\phi_\Pi = 1.5$, $\phi_y = 0.25/3$. In addition, let $\rho_d = \rho_z = 0.95^{1/3}$. Let $\rho_{\sigma_d} = \rho_{\sigma_z} = 0.75^{1/3}$. Let $\eta_d = \eta_d = 0.4$. And let $\gamma_d = -5$ and $\gamma_z = -6$. Last, set $\sigma_m = 0.25/1200$.

Exercises

A) Code the model.

B) Solve the model through a third-order perturbation.

C) Compute the *stochastic steady state*.

D) How does a monetary shock affect economic activity and inflation? Use a first-order approximation to check that the response looks right.

E) How does a demand level shock e_t^d affect economic activity and inflation? Use a first-order approximation to check that the response looks right.

F) Look at a 5 standard deviation shock to the volatility of the demand shock. Compute the generalized impulse responses starting at the stochastic steady state. Do this two different ways:

   i) by simulation.

   ii) using the closed-form formulae in Andreasen et al.

   How do your solutions to i) and ii) differ? Why?

G) How does the response in F) depend on the systematic response of monetary policy to the economy? Proceed in two steps.

   i) Define the term "natural rate of interest." How does the natural rate respond to the demand volatility shock? Explain the sign (economics!).

   ii) Look at a policy of strict inflation targeting. How does the economy respond to the volatility shock? Why is that?

If you want to read up a little, a good reference is Basu, Bundick (ECMA, 2017) "Uncertainty Shocks in a Model of Effective Demand."