Macroeconomics I PhD

Problem Set 5

2020/21

Please hand in at the latest February 3, 2021
`https://uni-bonn.sciebo.de/s/63uCG5GE28QTZag`

1 Fiscal Theory of the Price Level

This exercise is based on Leeper (1991). Consider a representative household who chooses the sequence of consumption, real money and real bond holdings, $\{c_t, m_t, b_t\}$,[1] so as to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{\log(c_t) + \log(m_t)\right\}$$

subject to the budget constraint

$$c_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} + \tau_t = y + \frac{M_{t-1}}{P_t} + R_{t-1}\frac{B_{t-1}}{P_t}.$$

The household has utility over real money holdings (MIU framework). In addition, there is no capital in this economy and the income/endowment y is exogenous and constant over time. The budget constraint is expressed in real terms (i.e. in the amount of consumption goods), while R_{t-1} is the *nominal* gross interest rate.

The government uses lump-sum taxes, τ_t, seigniorage (revenue from money creation), and debt issues to finance constant expenditures, g. The budget constraint of the government is

$$\frac{B_t}{P_t} + \frac{M_t}{p_t} + \tau_t = g + \frac{M_{t-1}}{P_t} + R_{t-1}\frac{B_{t-1}}{P_t}.$$

[1] $m_t := \frac{M_t}{P_t}$ and $b_t := \frac{B_t}{P_t}$. We also define gross inflation as $\pi_t := P_t/P_{t-1}$.

Goods market clearing ($c_t + g = y$) implies a constant consumption level of the households. The nominal interest rate and taxes are set by the government according to the monetary and fiscal rules specified below.

a) Derive the Euler equation for debt and the money-demand relation, (2.2) and (2.3) in Leeper (1991), from the necessary conditions of the household maximization problem.

b) Households are prevented from issuing debt, $B_t \geq 0 \, \forall t$. Formulate the household maximization problem as a sequence problem, and show that the transversality conditions for real balances and real debt,

$$\lim_{T\to\infty} \mathbb{E}_t \left\{\beta^T m_T\right\} = 0, \; \lim_{T\to\infty} \mathbb{E}_t \left\{\beta^T b_T\right\} = 0,$$

imply that sequences $\{m_s, b_s\}_{s=t}^{\infty}$ that fulfill the conditions derived in a) are optimal among all feasible sequences that start at some fixed values m_{t-1}, b_{t-1}.

Use the following proof method (cf. Stokey et al. (1989), section 4.5): given the sequence problem $\max_{\{x_s\}_t^\infty} \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} F(x_{s-1}, x_s)$, the sequence $\{x_t^*\}_t^\infty$, that fulfills the above conditions, and any feasible sequence $\{x_s\}_t^\infty$, show that

$$D := \lim_{T\to\infty} \mathbb{E}_t \sum_{s=t}^{T} \beta^{s-t} (F(x_{s-1}^*, x_s^*) - F(x_{s-1}, x_s)) \geq 0.$$

To do so, use that for any concave function f, $Df(a) \cdot (b-a) \geq f(b) - f(a)$, where $Df(a)$ is the Jacobian of f at a. Then, rearrange to find the term $F_2(x_{s-1}^*, x_s^*) + \beta \cdot F_1(x_s^*, x_{s+1}^*)$ in the sum, and go from there.

In the following, assume that the transversality conditions are fulfilled.

c) Show that in equilibrium, the present-value condition for real bonds holds, i.e.

$$b_t = \sum_{j=0}^{\infty} \mathbb{E}_t \left\{\beta^{j+1} s_{t+j+1}\right\},$$

where $s_t := \tau_t - g + \frac{M_t - M_{t-1}}{P_t}$ is the government surplus.

Suppose nominal gross interest rates are set according to

$$R_t = \alpha_0 + \alpha \pi_t + \theta_t,$$

where θ_t is white noise (i.e. a zero mean, iid process). Further suppose taxes are set according to

$$\tau_t = \gamma_0 + \gamma b_{t-1} + \psi_t,$$

where ψ_t is white noise. $\alpha_0, \alpha, \gamma_0, \gamma$ are policy parameters chosen by the government.

d) Derive the following linearizations around the zero-inflation steady state, assuming for simplicity that $\bar{b} = 1$[2]:

$$\mathbb{E}_t \widehat{\pi}_{t+1} = \alpha\beta\widehat{\pi}_t + \beta\theta_t, \tag{1}$$

$$\varphi_1\widehat{\pi}_t + \widehat{b}_t + \varphi_2\widehat{\pi}_{t-1} + (\gamma - \beta^{-1})\widehat{b}_{t-1} + \varphi_3\theta_t + \psi_t + \varphi_4\theta_{t-1} = 0. \tag{2}$$

Spell out $\varphi_1, \varphi_2, \varphi_3, \varphi_4$.

e) Derive the conditions for a (locally) unique equilibrium, ($|\beta^{-1} - \gamma| < 1$ and $|\alpha\beta| > 1$) or ($|\beta^{-1} - \gamma| > 1$ and $|\alpha\beta| < 1$). Assume $\gamma \geq 0$ and $\alpha \geq 0$. Provide a brief intuition for the conditions.

Possible solution path: cast the system in the form $A\,\mathbb{E}_t\, x_{t+1} = Bx_t + C\epsilon_t$ *with the endogenous variables* $x_t = [\mathbb{E}_{t-1}\,\widehat{b}_t, \mathbb{E}_{t-1}\,\widehat{\pi}_t, \widehat{\pi}_t - \mathbb{E}_{t-1}\,\widehat{\pi}_t]$. *Derive analytically its generalized eigenvalues, and argue with the Blanchard-Kahn conditions.*

f) For the active monetary policy case (the first set of conditions), derive the law of motion of $\widehat{\pi}_t$ and $\widehat{R}_t$. Would it be feasible to uncover monetary policy parameter α by regressing observed nominal interest rates on inflation?

[2] As before, $\widehat{x} = \frac{x - \bar{x}}{\bar{x}}$ denotes the relative deviation from steady state.

For the numerical part, set $\beta = 0.99$, $y = 0.5$, $g = 0.1$, and consider the policy rules

$$R_t = \beta^{-1} + \theta_t, \tag{3}$$

$$\tau_t = 0.11 + \psi_t,\ \psi_t = 0.99 \cdot \psi_{t-1} + \epsilon_{t-4}^{\psi,4} + \epsilon_t^{\psi} \tag{4}$$

where $\epsilon_t^{\psi,4}$ is a news shock about future taxes with an anticipation horizon of 4 periods.

g) Compute the responses of real bond holdings, real balances, and inflation to a monetary policy shock that raises the nominal interest rate by 1%. Interpret the results. *Hint: Using Klein's method, you can treat exogenous shocks as endogenous states, if needed.*

h) Compute the responses of real bond holdings and inflation to an anticipated 1% tax increase (transitory, but persistent) 4 periods from now. Interpret the results. *Hint: To implement the news shock, add 4 state variables to the system that "track" the news over time.*

References

Leeper, E. M. (1991). Equilibria under 'active' and 'passive' monetary and fiscal policies. *Journal of Monetary Economics 27*, 129–147.

Stokey, N. L., E. C. Prescott, and R. E. Lucas (1989). *Recursive methods in economic dynamics.* Harvard University Press Cambridge, Mass.