

# International Economics (Master) - Problem Set 3

## General remarks:

Problem set 3 will be discussed in the exercise session on May 24. Submit your solutions on ecampus by May 23, 2 p.m.

## Exercises

The exercise is meant to help you

- understand dynamics of the trade balance
- and see in a pencil and paper example how endogenous capital accumulation can help get the similar dynamics as in the data.

### **1. Productivity shocks and the trade balance**

Consider the following three-period model of an open economy. Periods are marked  $t = 0, 1, 2$ . A representative household maximizes life-time utility

$$\ln(c_0) + \ln(c_1) + \ln(c_2),$$

subject to the period budget constraints

$$c_t + k_{t+1} + d_{t-1} = y_t + d_t, \quad t = 0, 1, 2.$$

$c_t$  marks consumption in period  $t$ ,  $k_{t+1}$  marks the capital stock for the next period.  $y_t$  marks output in  $t$ ,  $d_t$  marks borrowing from abroad, to be repaid next period. We assume that the interest rate on borrowing is zero. Note that the formulation above reflects that capital is assumed to fully depreciate after having been used in production. Output is produced according to

$$y_t = A_t k_t^\alpha, \quad \alpha \in (0, 1)$$

$k_0 > 0$  is given.  $d_{-1} = 0$ .  $A_t$  is productivity in period  $t$ . At the beginning of period 0 (before making decisions about  $c_0, k_1, \dots$ ) the household learns the values of  $A_0, A_1, A_2$ . Observe that the market will impose  $d_2 \leq 0$ .

1. Set up the household's Lagrangian.
2. Derive the first-order conditions.

—— In the following, assume  $\alpha = 0.5$ ,  $k_0 = 1$  (so as to simplify expressions). ——

3. Express  $c_0, c_1, c_2$  as functions  $A_0, A_1, A_2$  only.

4. Express the period 0 trade balance ( $tb_0$ ) as functions of  $A_0$ ,  $A_1$ , and  $A_2$  only.
5. Suppose that  $A_0 = 2 \cdot (1 + x)$ ,  $A_1 = 2 \cdot (1 + \rho x)$ ,  $A_2 = 2 \cdot (1 + \rho^2 x)$ , with  $0 \leq \rho < 1$ . Derive

$$\left. \frac{\partial tb_0}{\partial x} \right|_{x=0},$$

as a function of  $\rho$ . (That is, what is the effect of the trade balance of a marginal increase in productivity?)

6. Provide the sign

$$\left. \frac{\partial tb_0}{\partial x} \right|_{x=0}$$

How does it depend on  $\rho$ ? Why is that? Provide the economics (doing so, you may find it useful to also define permanent income).

The purpose of the next exercise is to solve the open-economy RBC model on your own.

- practice setting up the model.
- see how well the model fits your favorite country.

## 2. The open-economy RBC model.

Take the open-economy RBC model discussed in class.

1. Calibrate the model to the economy for which you derived business-cycle statistics in the first problem set.
2. Provide the resulting parameters, a table for the steady-state values, and compare filtered second moments of the model to the data. Does it fit well.

—— Next, modify the model ——

3. So far, we have always assume a one-period time to build of capital. Now, instead, assume that there it needs to periods to build the capital stock, so that

$$k_{t+2} = (1 - \delta)k_{t+1} + i_t$$

Use the same parameter values as before. How does the propagation of the TFP shock change? Explain.

4. Assume that period utility is given by  $u(c_t, h_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \kappa \frac{h_t^{1+\gamma}}{1+\gamma}$ ,  $\sigma > 0, \kappa > 0, \gamma \geq 0$ . Calibrate this model to "your" economy. How well does the model fit the data? What explains the change in fit?